The Role of Memory in Mathematical Ability

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Ed.M., Harvard Graduate School of Education (2012)
Unpublished manuscript, submitted as coursework, December 2011

How does memory interact with mathematical ability? Is memory an improvable skill or a limited resource? Is there potential cognitive value to be derived from digit memorization, which could, in turn, influence a student's success in the mathematics classroom? The author establishes the narrative that developing the skill of digit memorization improves working memory, a critical contributor to mathematical performance, by enhancing its ability to engage in retrieval from long-term memory. He then posits a competing narrative which presents memory as a limited resource that should be employed pragmatically in mathematical problem solving, and compensated for, when necessary, by other available strategies. He ultimately argues for an understanding of the role of memory in mathematical ability that affords a measure of educational legitimacy to the popular but peculiar activity of memorizing the digits of pi.

I. Introduction.

The ratio of a circle's circumference to its diameter has been a subject of inquiry and of great fascination for thousands of years. This number, known as *pi* since the eighteenth century, has inspired advances in pure mathematics, in supercomputing design, and in demonstrations of the capabilities of human memory. While today's computer scientists have derived more than 10 trillion of the number's string of random, nonrepeating digits (Cornish, 2011), students in today's mathematics classrooms are memorizing dozens, hundreds, and even thousands of them.

Recitation of pi from memory is widely encouraged by teachers as part of the celebration of Pi Day, an annual holiday first observed in 1988 whose date, March 14, corresponds to the number's first three digits: 3.14 (Bialik, 2011). Teachers at all grade levels have embraced the occasion as a chance to break from routine and ignite in their students a fresh curiosity and excitement for mathematics. "Pi Day is a perfect day to celebrate mathematics," explained Eric Willis, a 7th grade teacher in Villa Park, Illinois. "It gave the students the opportunity to have fun while investigating mathematical concepts, and [to] be a little goofy" (Zeman, 2005).

Figure 1. Digits of Pi Recited from Memory: Largest Number Recorded per Grade Level, c. 2011

π = 3.14159265358979323846264338327950288419 71…

Grade	K	1st	2nd	3rd	4th	5th
# Digits	30	101	56	101	121	335

Grade	6th	7th	8th	9th	10th	11th
# Digits	2,522	3,310	1,004	690	2,990	10,980

(Anderson, n.d.; Barnes, 2011; Belden, 2007; Negley, 2011)

While festivities often include crafts, songs, storytelling, and treats, these memory challenges have become the cornerstone of Pi Day celebrations. "It's not like I'm in love with pi or something," said Dustin Foster, an 11th grade student in Tacoma, Washington who learned and recited the number's first 270 digits. "I feel like I can do it. It's something really big" (Modeen, 2004).

As this seemingly whimsical classroom activity grows in popularity, interesting questions arise about its educational value. How does memory interact with mathematical ability? Is memory an improvable skill or a limited resource? Is there potential cognitive value to be derived from digit memorization, which could, in turn, influence a student's success in the mathematics classroom? In response to these questions, I will establish the narrative that developing the skill of digit memorization improves working memory, a critical contributor to mathematical performance, by enhancing its ability to engage in retrieval from long-term memory. I will then posit a competing narrative which presents memory as a limited resource that should be employed pragmatically in mathematical problem solving, and compensated for, when necessary, by other available strategies. I will ultimately argue for an understanding of the role of memory in mathematical ability that affords to this popular but peculiar activity a measure of educational legitimacy.

II. The Educational Value of Digit Memorization

The idea that encouraging a student to memorize digits of pi could be beneficial to that student's classroom performance is not an immediately intuitive one, and likely has very little to do with the presumably lighthearted motives held by teachers who promote this activity. In this section, however, I will draw upon frameworks and findings from neuroscience and educational psychology to argue for a meaningful relationship between digit memorization and mathematical aptitude. Memory is a skill that can be strengthened with practice, and its enhancement can reap benefits for the mathematics student by contributing to the efficiency of working memory in its important role in mathematical ability.

Skilled Memory Theory

The first question that must be addressed is whether one's capacity for remembering numbers can in fact be expanded. The digit span test, a measure of the number of digits that can be repeated accurately by a subject after they are briefly viewed or heard, is generally considered to be a test of short-term memory. The stubbornly limited capacity of short-term memory to hold no more than approximately seven unrelated items (e.g., Miller, 1956), however, presents an apparent contradiction when considering the many documented subjects with exceptional memories for numbers (Ericsson et al., 1980).

An attempt to resolve this incongruity was depicted in a 1980 study by Ericsson et al., in which a college student of average memory abilities and intelligence engaged in 230 hours of practice of the digit span task over the course of 20 months. Remarkably, as his practice time accumulated, his digit span steadily increased from seven digits to 79. The authors determined that he achieved this dramatic result by devising mnemonic associations with subgroups of three or four digits, storing these subgroups in his long-term memory, and developing structures for retrieving them effectively. They concluded that it is not possible to increase the capacity of short-term memory itself, but that the use of long-term memory for storing mnemonic associations, coupled with the employment of effective retrieval structures and repeated practice, can lead to dramatic expansion of one's memory skills.

Replication of these results and further work by Ericsson and his colleagues in the 1980s led to the development of *skilled memory theory* (Chase & Ericsson, 1981, 1982; Ericsson, 1985, 1988; Ericsson & Chase, 1982; Ericsson & Faivre, 1988; Ericsson et al., 1980; as cited in Takahashi et al., 2006). This framework proposed three principles for building an exceptional memory, whether for digits, words, or practical skills in which expertise can be developed, such as taking restaurant orders (Ericsson & Polson, 1988, as cited in Thompson et al., 1991): a) *meaningful encoding*, the involvement of preexisting knowledge and experiences in devising mnemonics or otherwise encoding new information; b) *retrieval structure*, the explicit attachment of cues for accessing the information later; and c) *speed-up*, the reduction in the time required for these operations through practice.

Skilled memory for numbers may require extensive practice, but it does not require superior cognitive abilities. Takahashi et al. (2006) administered a variety of memory tasks to Hideaki Tomoyori, who at age 54, in 1987, recited 40,000 digits of pi, which constituted a world record at the time. In achieving this feat, Tomoyori employed a mnemonic system, based on features of the Japanese language, which he had developed through practice. The authors studied performance on the same array of tasks by a control group, matched for age and educational attainment. They found that Tomoyori's results were generally strong on the digit-related tasks, but not markedly exceptional in situations where it was not feasible for him to apply his mnemonic strategies. His results on the verbal tasks (i.e., recalling word lists and short stories) were not at all remarkable, despite his well-developed use of words and sentences as retrieval cues for digits. The authors deemed their findings to be consistent with Ericsson's skilled memory framework, concluding that a superior memory is merely a skill developed through practice, requiring no prerequisite intelligence and implying nothing about aptitude in other unpracticed tasks. In this way, the task of memorizing digits of pi is a widely accessible one.

Working Memory and Mathematical Ability

The construct known as *working memory* is a limited capacity resource that concurrently preserves and processes information (Berg, 2008). Its traditional model consists of three components: a control system known as the *central executive*, and two storage systems, the *visuospatial sketchpad* and the *phonological loop*. A later iteration of the model has incorporated a fourth component, the *episodic buffer*, which integrates information from the two storage systems into episodes and interfaces with long-term memory (Baddeley, 2003).

Measures of working memory have been found to have meaningful predictive power in cognitive and educational measures in children. Working memory tasks are, according to Alloway & Passolunghi (2011), a "pure measure of a child's learning potential," and working memory skills are "able to predict a child's performance in learning outcomes" (p. 134). Their study of 206 normally developing 7- and 8-year-old children aimed to dissociate working memory from verbal skills and measures of knowledge that a child has already learned (e.g., the IQ test). They found that even when age and vocabulary differences were taken into account, "memory skills uniquely predicted mathematical skills and arithmetical abilities" (p. 136).

Numerous other studies in recent years have investigated and substantiated the relationship between working memory and mathematical performance (Alloway, 2006; Andersson, 2007; Berg, 2008; Camos, 2008; Logie et al., 1994; Raghubar et al., 2010; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004; Swanson & Kim, 2007). In fleshing out this relationship, researchers have differentiated between the contributions of working memory's subcomponent systems, and identified its impact on performance in various mathematical tasks.

All three subcomponents of the traditional model of working memory have been demonstrated to contribute to mathematical performance. Mathematical skills are particularly closely linked to visuospatial sketchpad, which serves as a "mental blackboard" upon which numbers are represented during counting and assignment of place value (Alloway, 2006). Visuospatial memory has been shown to contribute unique variance to mental calculation in children (Berg, 2008), and found to be employed heavily by adults who possess strong mental calculation abilities (Hatano & Osawa, 1983). Measures of both the phonological loop and the central executive have also been found to contribute uniquely to mathematical word problem solving in children, as seen in a study of 2nd, 3rd, and 4th graders which controlled for the influence of reading ability, age, and IQ (Andersson, 2007). Finally, a 1994 study by Logie et al. presented a framework in which all three components of working memory operate in tandem to support different aspects of the complex task of mental calculation.

A closer look at mental calculation highlights the importance of a child's working memory not only as an enabler of her mathematical skills, but also as a limitation on them. Calculation is a building block of mathematical ability; the National Council of Teachers of Mathematics in 2006 underscored the importance of promoting calculation abilities in the early school years (Berg, 2008), as a precursor to developing more complex skills. Mental calculation involves the deployment of working memory's resources, as seen in the Logie et al. study (1994), but it is also bounded by working memory's limits. When arithmetic is too challenging to do "in one's head," it is because "the storage demands of the activity exceed the capacity of working memory" (Alloway, 2006, p. 135). This prevents a student from *processing* a computation for which he may be entirely familiar with the concepts and operations required.

The capacity of a child's working memory can become a bottleneck in his ability to achieve success in the mathematics classroom. Given the crucial role of working memory, potential avenues for expanding its functionality must be considered.

Long-Term Memory as Facilitator of Working Memory

Given its inherent constraints of limited and temporary storage, how might working memory take greater advantage of its access to the information stored in long-term memory? Is there something valuable to be learned from the skilled memory framework?

A key function of the central executive subcomponent of working memory is the accessing and retrieving of knowledge from long-term memory (Andersson, 2007). As a student engages in a word problem, for example, she may call upon her long-term memory to retrieve arithmetic rules, facts, or other task-relevant information. The greater the information stored in long-term memory and the more rapidly it can be accessed, the more effectively a student's working memory can serve in solving problems.

Efficient retrieval from long-term memory is a skill that is capable of being enhanced through practice, according to skilled memory theory, and long-term memory is not bound by the capacity limits of short-term memory. Optimized access to a deep store of information in long-term memory would prove quite valuable as support for working memory in action. Ericsson, the architect of skilled memory theory, worked with Kintsch (1995) to propose an expansion of the general account of working memory to incorporate this valuable function. They called for the inclusion of "a mechanism based on skilled use of storage in long-term memory" (p. 211), calling this function *long-term working memory*. Information in long-term working memory is both stable and durable, but it can only be activated temporarily. Activation is achieved by establishing retrieval cues by which one draws the desired information into working memory.

This system of tapping into long-term storage via short-term cues is precisely what emerges when a person develops mnemonics in order to expand his digit span, or to memorize pi. If the skill of moving information rapidly across this cognitive bridge becomes highly developed, perhaps through practice at digit memorization, then working memory stands to benefit greatly from the enhanced flow of stored knowledge at the very moment it is needed.

Conclusion

In developing skilled performance abilities, as in digit memorization and mental calculation, students can acquire the myriad benefits of enhanced working memory functionality (Ericsson & Kintsch, 1995). With practice, long-term memory can become the home for an ever-expanding quantity of useful but more obscure mathematical facts like “15-squared equals 225” (Rickard et al., 2008). The efficient retrieval of these facts, developed via skilled memory pursuits like digit memorization, can ultimately speed up mental calculation, reduce the novelty of new problems faced, improve recognition of appropriate methods and procedures, and contribute to success in the mathematics classroom.

III. Exploring a Competing Narrative: Focus on Flexible Strategy Use

Memorization challenges on Pi Day are surely an amusing departure from the routine, but if a teacher wants to encourage success in his mathematics classroom, his focus should remain squarely on helping his students to develop habits of choosing wisely among the various problem-solving tools at their disposal. There are multiple routes through the brain for mathematical problem solving, and memory, which supports some but not all of them, is best viewed within a framework of flexible strategy use—as a limited resource to be employed pragmatically, and compensated for, when necessary, by other available strategies.

Questioning Skilled Memory Theory and the Role of Long-Term Memory

Not all world-famous memorists fit neatly into the skilled memory framework. In fact, the success of another pi memorizer calls into question the necessity of one of the three components upon which Ericsson’s model is based. Before presenting an argument to reframe memory as only one aspect in a flexible approach to problem solving, I will take issue with the soundness of skilled memory theory, and the role of long-term memory in working memory.

Immediately preceding Tomoyori in the record books was a man named Rajan Mahadevan, who, in 1981, recited 31,811 digits of pi in just under four hours. Thompson et al. (1991, 1993) documented Mahadevan’s approach to the memory feat and subjected him and a control group to experiments involving typical memory tasks. In applying the skilled memory framework, they confirmed his use of retrieval cues, and his ability to speed up his memorization with practice. They concluded, however, that Mahadevan made very little use of the third component: meaningful encoding. Skilled memory theory suggests that in the absence of mnemonic devices or other methods for relating number sequences to one’s pre-existing knowledge, a normal subject’s digit span will remain at about seven items (Thompson et al., 1991). However, Mahadevan achieved a digit span of 60 digits without engaging in meaningful encoding, leading the authors to deem his exceptional memory capacity to be innately endowed, at least in part. In so doing, they challenged the theory’s hypothesis that the acquisition of a skilled memory, which requires engagement of all three stated components, is the only means by which any person can overcome the rigid limits imposed by short-term memory.

Because skilled memory theory is the foundation on which the construct of long-term working memory is built, we must take pause and ask whether the link between long-term memory and working memory is a developable skill after all. In fact, one’s control over the quantity and the reliability of the information that moves into long-term storage from short-term memory has been shown to be limited (Anderson, 1983; Craik & Lockhart, 1972, as cited in Ericsson & Kintsch, 1995). It is not clear that access to long-term memory can be the engine for directed improvement in a working memory task like mathematical problem solving.

Multiple Routes for Mental Arithmetic

To focus solely on memory’s role is to overlook other fundamental aspects of how mathematical problems are actually solved. In fact, an investigation of the cerebral pathways involved in performing mental calculation reveals that sometimes, memory is not involved at all.

Mental calculation can take one of two routes through the brain, according to a study by Dehaene and Cohen (1995): a “direct” route, which appeals to rote verbal memory of stored familiar facts, or an “indirect” route, which encodes the numbers into quantities and processes them purely as calculations. The authors drew this conclusion after comparing the particular computational disabilities of a patient with a left subcortical lesion and another patient with a right inferior parietal lesion, and establishing that a double dissociation existed between rote verbal knowledge and mechanical quantitative knowledge.

Notably, Dehaene and Cohen demonstrated that the indirect route, which requires visual and magnitude coding to support the manipulations required for calculation, is called upon only when rote verbal memory is lacking, i.e., when no stored facts exist that can provide a shortcut. While perhaps intuitive, this evidence is reflective of a basic notion of pragmatism in the brain, as it seeks to minimize the time and/or effort required to produce a solution. When asked to compute 9 times 12, one person might recall it instantly from her multiplication tables, another might recall that 9 times 6 is 54 and take an extra moment to double it, and a third person, at a loss for a shortcut, might begin with 12 and add 12 eight more times in succession. The brain appears to allow for and encourage this kind of flexibility in solving mathematical problems.

The pathways need not be geographically distinct for the notion of competing pathways to hold. Another study used functional MRI scans of nine adult subjects during the acts of basic fact retrieval and of manual calculation, and found that the areas of cortical activity were similar, but that cortical activation was higher and geographically broader in the calculation task (Kazui et al., 2000). Regardless of geographical precision, this pragmatic variability in brain pathways during mathematical activity is best utilized, as I will argue presently, by flexibility in strategy.

Flexible Strategy Use: Method over Mind

Success in mental mathematical performance is driven primarily by method of approach, as opposed to one's absolute level of working memory strength or capacity. This is simply because the limits of a child's working memory can be accounted for and circumvented when the choice of his problem solving strategy is up to him. In a study by Hope and Sherrill (1987), 15 skilled and 15 unskilled students were selected from a large high school population based on their performances on a mental multiplication test, and their individual methods and strategies were then examined and compared. The mastery of basic facts, like the kind that would be retrieved by rote, was not found to be a differentiator in terms of mental calculation performance. Furthermore, there was only a weak direct relationship between mental calculation ability and short-term memory capacity, as reflected in the digit span test. The authors suggested that their findings may reinforce earlier published arguments that flexibility in calculation strategy, which the subjects were afforded here, may be the equalizer between different students with widely varying working memory capacities (p. 110):

> "[T]he resourceful person can always find a way to ease the memory burden by selecting a strategy that requires little information processing. Through the judicious selection of a calculative strategy, a skilled calculator can get by with fewer short-term memory resources than the selection of more inefficient strategies would necessitate."

Even the intuition that the best problem solvers must draw their strength from powerful memories is not without dissent. The methods of a 13-year-old mental calculation expert were studied in detail by Hope (1987), who determined that measures of memory capacity and digit span were far too rudimentary to explain her skills. "I'm not very good at memorizing," the child explained (p. 339), though her digit span was found to be in the 95th percentile. The author concluded that success in mental calculation depends primarily upon the ability to "select the right tool for the job" (p. 339).

The ability to estimate answers to prohibitively complex computations, another important developmental skill, is also primarily a function of the strategies known and employed, and not a function of use of or strength in memory capacity, according to a study of the selected estimation methods of 44 subjects with no particular strength in mental calculation. In the study, Dowker (1992) presented evidence that "flexible strategy use is related to success in estimation, in mathematics in general, and in many cognitive tasks" (p. 53).

Conclusion

Mathematical ability is primarily a function of one's recognition and deployment of the optimal method for deriving a solution given environmental constraints, including the strength of one's memory. The brain has multiple ways of arriving at a mathematical answer, only one of which is via the retrieval of facts from long-term memory. To be sure, there is something to be gained by strengthening the practical value of memory, especially in its store of basic mathematical facts that may come in handy as strategic shortcuts, but the service provided by one's memory is best viewed merely as one aspect of a varied problem-solving toolkit.

IV. Discussion

In this section, I will address certain portions of the competing narrative, and incorporate its most valuable elements into a more comprehensive account of the capacity for educators to improve upon a student's mathematical ability.

In Defense of Skilled Memory Theory

Thirteen years after being challenged by Thompson et al. (1991) with respect to the failure of skilled memory theory to account for the allegedly innate memory skills of Rajan Mahadevan, Ericsson et al. (2004) brought Mahadevan into their own laboratory, interviewed and tested him extensively, and concluded that his case was not an exception to the rule at all.

Core to the 1991 findings, as described in the counterargument, was the claim that Mahadevan needed no mnemonic associations or other forms of meaningful encoding during his digit memorization. They highlighted his ability to "chunk" digits into subgroups of 10 to 15 as his basic units of storage, far more than the 3 to 5 digit groups used by other memory experts. This, they had argued, revealed that he was endowed with a superior short-term memory capacity, upon which he had built additional skills to enhance his performance—and that only in his incremental skills were his efforts consistent with the principles of skilled memory theory.

Ericsson et al. (2004) reconsidered the evidence and logic behind this argument, and with the addition of their own primary research and a closer look at Mahadevan's past, concluded that his purportedly innate excess capacity was nothing more than the result of ardent practice. Well before becoming a formal research subject, he had spent roughly 1,000 hours over eight years memorizing 30,000 to 40,000 digits of pi, and in so doing had organically developed encoding techniques so refined that they had become almost seamless in small sequences of numbers. The ability to develop this kind of advanced encoding was also consistent with their account of the powerful role of long-term working memory. Ericsson et al. acknowledged the uniqueness of Mahadevan's lifelong devotion, but found no inconsistency in his story with the notion that exceptional memory for numbers can only be learned, and must be acquired through practice.

From this defense of Ericsson's theory, I derive greater confidence in my view that skilled memory pursuits like digit memorization build the muscle necessary to support greater passage between working memory and long-term memory, and thus improve working memory's ability to facilitate the problem solving skills that determine a student's success in the classroom.

Finding Common Ground

With the resilient construct of skilled memory theory firmly in hand, we can move on to the task of finding meaningful common ground in the two arguments presented here. While I have argued firmly for the expansion of working memory through skilled memory acquisition as a valuable contributor to mathematical ability improvement, there is also merit in the counterargument's emphasis on developing a variety of pathways for solving problems and improving one's recognition of the right tool or technique at the right time. Because a teacher's time with students is scarce and success is so important, we need to determine whether a binary choice must be made between these two approaches, training memory and training strategy, or if it is a false dichotomy. Fortunately, I find that my argument and the counterargument are far from mutually exclusive, and an evolving conversation in the research literature has the potential to weave them into a unified narrative.

This convergence becomes evident when the counterargument's two distinct problem solving pathways, representing manual calculation and memory retrieval, are viewed to be in competition for the fastest and most efficient production of the correct answer. There is a growing body of research on this race between the two methods, including what tips the scale toward one approach or the other, and at what point this choice is made (Bajic & Rickard, 2011). At the foundation of this work is a consensus view, according to Rickard et al. (2008), that with repeated exposure to a given type of mathematical problem, there is a shift toward the use of *retrieval*, and away from the several steps involved in manual calculation. In this way, *practice* of a skill is leading to the selection of *memory* as the most efficient *strategy*. Debates continue about the precise timing and mechanics of this phenomenon (Bajic & Rickard, 2011), but the common ground is clear: memory, as a tool in flexible strategy use, becomes an increasingly attractive option as it expands with practice.

Educational Implications

The embrace of a model in which mathematical ability can be improved from both ends, through the building of memory skills and through the guiding of thoughtful strategy selection, offers great potential for teachers. Mathematics education research to date appears to have internalized the value of strategy selection; several of the studies supporting this argument came from the *Journal for Research in Mathematics Education* (Dowker, 1992; Hope, 1987; Hope & Sherrill, 1987). But the groundswell of evidence for working memory's role in mathematical skills, and for the value of memory building itself, may not be as present in the minds of the subject area's own education researchers, curriculum designers, or classroom instructors. Against this backdrop, I offer three basic recommendations to those who write curriculum or teach it in the classroom.

First, mathematics teachers should engage students in tasks that seek to build working memory itself. A useful way of framing and organizing the many options for pursuing this goal can be borrowed from the current discourse on the *training* of working memory. Morrison and Chein assessed the state of this emerging field in 2011, and sorted its rapidly growing collection of studies into two broad approaches that they identified as *core training* and *strategy training*. Core training involves demanding working memory tasks intended to enhance domain-general working memory mechanisms, while strategy training promotes the use of domain-specific strategies, including encoding and retrieval, to support the retention of information over time. As the cognitive benefits of these training regimens are evaluated, researchers are paying particular attention to their transferability. Early results have shown core training, by its nature, to have far-reaching transfer effects, and while transfer from strategy training is less certain, one study found that working memory strategy training in schoolchildren led to improvements in mental calculation (St. Clair-Thompson, 2010, as cited in Morrison & Chein, 2011).

Second, the teaching of problem solving strategies and techniques should incorporate an increased focus on hands-free mathematics. In an age where calculators appear on every personal electronic device, it may be easy to overlook the value of developing mental calculation techniques and of adding to the store of basic facts that stand ready to be accessed in long-term memory. Likewise, the focus on acquisition of manual written procedures, like long division techniques, should be assessed with respect to their contribution to a student's lasting skill set and his cognitive development. As I have established here, mental problem solving, in both calculation and retrieval modes, is a skill whose development will have an enduring impact.

Third and finally, teachers should encourage in their students the pursuit of skilled memory. Digit memorization need not be confined to Pi Day; they can start very practically by presenting a 2-by-2 grid of numbers for a few seconds, and then asking the class to transcribe it from memory. Over time, they can build to grids of 3-by-3 and even 4-by-4. Increasingly challenging digit span tasks, as they begin to exceed the limits of pure short-term memory, will spur students' development of both meaningful encoding and retrieval structures.

Teachers can foster meaningful encoding directly by encouraging students to memorize numbers that have personal meaning to them, like the jersey numbers and statistics of their favorite athletes, or the birthdays of their friends and relatives. The student whose dramatic increase in digit span provided Ericsson et al. (1980) one of their earliest glimpses of the skilled memory framework was a runner, and they found that the majority of his mnemonic associations were based on running times for various races. Systematic retrieval of these encoded subgroups will soon follow, as seen in the approach of Sierra Van Such, who recited an astounding 1,266 digits of pi in the 7th grade: "I just take little groups of numbers and try to find patterns in the big numbers and then just keep putting them together" (Newport, 2011). With practice, students will begin to develop a skilled memory, and to reap the cognitive and educational rewards.

Open Questions and Next Steps

In closing, I will briefly identify two broad questions whose addition to the research literature would advance the understanding of this subject, and would either challenge or confirm my argument and my recommendations for classroom implementation.

The first task for researchers would be to test the relationship between digit memorization and mathematical ability more directly, thus bypassing the intermediate step of working memory. Working memory is merely a construct, and an evolving one, and there is a greater chance for error in the logic and in the resulting educational conclusions when the argument for a link between memory skill and classroom performance can only be established as an inference based on the link of each to working memory. Study design is rather intuitive here; one could test the mental calculation skills of a class at the beginning and end of an academic year, and subject half of the class to periodic digit span training outside of class time during the year. Once a positive relationship is established, further work could be done on the type and frequency of practice that is most effective, similar to the research being done on spacing (e.g., Rickard, 2008).

To establish a more complete picture of the relevance of memory to mathematical learning, the other key avenue to be explored is the relationship between the various memory functions and the increasingly abstract mathematical concepts that students confront as they mature. While the more tangible skill of mental calculation has largely been the focus both in this discussion and in the research literature, we must pursue the question of whether memory's impact ends with the mechanical tasks of arithmetic, or whether it can either directly or indirectly empower a student as she encounters concepts in algebra, geometry, and beyond. Studies in this space would require great collaboration between neuroscience and education researchers, as they seek to identify pathways and measure performance for far less definable cognitive tasks.

Nonetheless, I am encouraged by the research performed to date in each respective discipline, intrigued by the potential for its interdisciplinary convergence, and emboldened to promote the activity that plays a tongue-in-cheek role on Pi Day, but that just might have broader benefits for the teaching and learning of mathematics.

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